Vom Differenzenquotient zum Differentialquotient

Beispiel: $f(x) = \frac{1}{20}x^2 + \frac{1}{10}x + 4$

$$m_{Sekante} = \frac{\Delta y}{\Delta x} = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{f(x_0 \pm h) - f(x_0)}{(x_0 \pm h) - x_0}$$

Differenzen - quotient

$x_0 = 2 \qquad f(x_0) = 4\,{}^2/_5$

$f(x_0 - h) = {}^1/_{20}\,(2 - h)^2 + {}^1/_{10}\,(2 - h) + 4$

$$m_{Sekante} = \frac{\frac{1}{20}(2-h)^2 + \frac{1}{10}(2-h) + 4 - 4\frac{2}{5}}{(2-h) - 2}$$

$$m_{Sekante} = \frac{\frac{1}{20}(4 - 4h + h^2) + \frac{1}{10}(2-h) - \frac{2}{5}}{-h}$$

$$m_{Sekante} = \frac{-\frac{1}{5}h + \frac{1}{20}h^2 - \frac{1}{10}h}{-h}$$

$$m_{Sekante} = \frac{3}{10} - \frac{1}{20}h$$

Grenzwertübergang: x nähert sich von links an x_0

$$m_{Tangente} = \lim_{h \to 0} \left(\frac{3}{10} - \frac{1}{20}h\right) = \frac{3}{10}$$

$x_0 = 2 \qquad f(x_0) = 4\,{}^2/_5$

$f(x_0 + h) = {}^1/_{20}\,(2 + h)^2 + {}^1/_{10}\,(2 + h) + 4$

$$m_{Sekante} = \frac{\frac{1}{20}(2+h)^2 + \frac{1}{10}(2+h) + 4 - 4\frac{2}{5}}{(2+h) - 2}$$

$$m_{Sekante} = \frac{\frac{1}{20}(4 + 4h + h^2) + \frac{1}{10}(2+h) - \frac{2}{5}}{h}$$

$$m_{Sekante} = \frac{\frac{1}{5}h + \frac{1}{20}h^2 + \frac{1}{10}h}{h}$$

$$m_{Sekante} = \frac{3}{10} + \frac{1}{20}h$$

Grenzwertübergang: x nähert sich von rechts an x_0

$$m_{Tangente} = \lim_{h \to 0} \left(\frac{3}{10} + \frac{1}{20}h\right) = \frac{3}{10}$$

h → 0

$$m_{Tangente} = \frac{df}{dx}(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{(x_0 + h) - x_0} = \lim_{h \to 0} \frac{f(x_0 - h) - f(x_0)}{(x_0 - h) - x_0} = f'(x_0)$$

Differential - quotient