Musterlösung: **12. Jgst.** **3. Test** **Datum: 01.03.2005**

Klasse: GY 03 c **Fach: Mathematik (Leistungskurs)**

Thema: **Integralrechnung: Partielle Integration, Integration durch Substitution und Volumina von Rotationskörpern**

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❶ **Integration durch Substitution**

Ermitteln Sie eine Stammfunktion zu folgenden Integralen:

a)

$$\int xe^{2x^2-3}dx = \frac{1}{4}\int e^u du = \frac{1}{4}e^u + c = \frac{1}{4}e^{2x^2-3} + c$$

$$Substitution:$$

$$u(x) = 2x^2 - 3 \Rightarrow \frac{du}{dx} = 4x \Rightarrow \frac{1}{4}du = x \cdot dx$$

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b)

$$\int \sin(x)\cos(x)dx = \int u\, du = \frac{1}{2}u^2 + c = \frac{1}{2}\sin^2(x) + c$$

$$Substitution:$$

$$u(x) = \sin(x) \Rightarrow \frac{du}{dx} = \cos(x) \Rightarrow du = \cos(x)dx$$

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c)

$$\int \frac{\ln(x)}{x}dx = \int u\, du = \frac{1}{2}u^2 + c = \frac{1}{2}\left[\ln|x|\right]^2 + c$$

$$Substitution:$$

$$u(x) = \ln|x| \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow du = \frac{1}{x}dx$$

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d)

$$\int \frac{x^2}{a-6x^3}dx = -\frac{1}{18}\int \frac{1}{u}du = -\frac{1}{18}\ln|u| + c = -\frac{1}{18}\ln\left|a-6x^3\right| + c$$

$$Substitution:$$

$$u(x) = a - 6x^3 \Rightarrow \frac{du}{dx} = -18x^2 \Rightarrow -\frac{1}{18}du = x^2 dx$$

❷ **Partielle Integration**

Ermitteln Sie eine Stammfunktion zu folgenden Integralen:

a)

$$\int xe^x dx \quad = \quad xe^x - \int e^x dx \quad = \quad xe^x - e^x + c$$

$$g(x) = x \qquad h(x) = e^x$$

$$g'(x) = 1 \qquad h'(x) = e^x$$

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b)

$$\int x^2 \cos(x)\,dx \quad = \quad x^2 \sin(x) - 2\int x \sin(x)\,dx$$

$$g(x) = x^2 \qquad h(x) = \sin(x)$$

$$g'(x) = 2x \qquad h'(x) = \cos(x)$$

$$\int x \sin(x)\,dx \quad = \quad -x\cos(x) - \int \left[-\cos(x)\right] dx \quad = \quad -x\cos(x) + \sin(x)$$

$$u(x) = x \qquad v(x) = -\cos(x)$$

$$u'(x) = 1 \qquad v'(x) = \sin(x)$$

$insgesamt:$

$$\int x^2 \cos(x)\,dx \quad = \quad x^2 \sin(x) - 2\left[-x\cos(x) + \sin(x)\right]$$

$$= \quad x^2 \sin(x) + 2x\cos(x) - 2\sin(x) + c$$

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c)

$$\int \ln(x)\,dx \quad = \quad x\ln(x) - \int 1dx \quad = \quad x\ln(x) - x + c$$

$$g(x) = x \qquad h(x) = \ln(x)$$

$$g'(x) = 1 \qquad h'(x) = \frac{1}{x}$$

d)

$$\int \sin^2(x)\,dx = -\sin(x)\cos(x) + \int \cos^2(x)\,dx$$

$$g(x) = \sin(x) \qquad h(x) = -\cos(x)$$

$$g'(x) = \cos(x) \qquad h'(x) = \sin(x)$$

$$\int \sin^2(x)\,dx = -\sin(x)\cos(x) + \int \left[1 - \sin^2(x)\right] dx$$

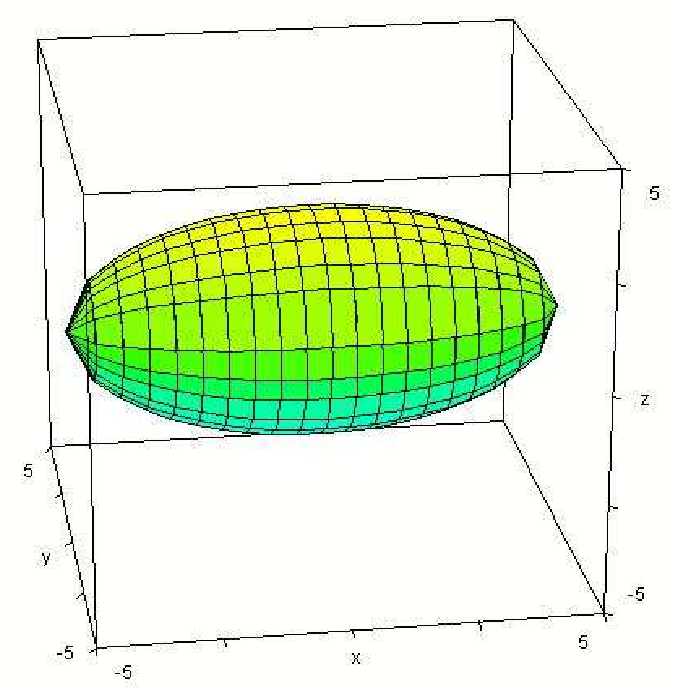
$$\int \sin^2(x)\,dx = -\sin(x)\cos(x) + \int 1\,dx - \int \sin^2(x)\,dx$$

$$2\int \sin^2(x)\,dx = -\sin(x)\cos(x) + x$$

$$\int \sin^2(x)\,dx = \frac{1}{2}\left[-\sin(x)\cos(x) + x\right]$$

❸ **Rotationsvolumina**

Ermitteln Sie das Volumen des Körpers. Die Randfunktion f(x) hat folgende Funktionsvorschrift:



$$f(x) = \frac{1}{2}\sqrt{25 - x^2}$$

Lösung:

$$V(x) = \pi \int_{-5}^{5} \left(\frac{1}{2}\sqrt{25 - x^2}\right)^2 dx = 2 \cdot \pi \cdot \frac{1}{4} \cdot \int_{0}^{5} \left(25 - x^2\right) dx$$

$$V(x) = \frac{1}{2} \cdot \pi \cdot \left[25x - \frac{1}{3}x^3\right]_0^5 = \frac{125}{3}\pi$$